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DYNAMIC RESPONSE OF NORMAL MOVING LOAD IN AN INITIALLY STRESSED TRANSVERSELY ISOTROPIC HALF-SPACE

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Abstract—In this paper the stresses developed in a transversely isotropic half-space under compressive prestressed conditions have been obtained due to a moving normal load on the rough surface. It has been shown that the existing prestresses have much effect on the development of incremental stresses due to normal load. The numerical calculations for developed stresses have been done for the case of ice for different values of prestressed parameters and have been presented in tabular form The condition under which the fracture on the surface will take place has also been derived, and it has been shown that the critical velocity at which the fracture will take place is also a function of initial stresses present in the body. The numerical values for the velocities of moving load for generating fracture have been calculated for different prestressed parameters in case of ice

INTRODUCTION

The response of moving normal load over a surface is a subject of investigation because of its possible application in determining the strength of a structure. The steady-state solution of the problem of moving normal load over an elastic half-space was given by Cole and Huth[1] and Craggs[2], who derived a relatively simple closed form solution, exhibiting a resonance effect at a critical load velocity, which in this case equals the velocity of Rayleigh waves. The problem considered by Cole and Huth[1] had been discussed previously by Sneddon[3] by a somewhat different method. However, Sneddon[3] treats only a particular case (the subsonic case). Stresses developed in a transversely isotropic elastic half-space due to normal moving load over a rough surface have been determined by Mukherjee[4]. The problem of moving load on a plate resting on an elastic half-space has been solved by Sackman[5] and Miles[6]. The relevant systematic approach towards the problem of moving load on a plate resting on an elastic half-space has been given by Achenbach, Keshava and Herrmann[7]. Mukhopadhyay[8] has studied the problem of normal moving load over a transversely isotropic layer lying on a rigid foundation.

The earth is an initially stressed medium. Due to slow process of creep, atmospheric pressure, gravity, difference of temperature, etc. stresses of large magnitude, called initial stresses or prestresses, are present inside the earth. It is of interest to study the effect of the initial stresses while studying the problems of earth model.

The present paper is concerned with the effect of the prestresses on the generation of incremental stresses leading to fracture due to a normal moving load on the rough surface of the transversely isotropic elastic half-space. The stresses have been obtained in closed form, and numerically the values have been calculated for different magnitudes of the prestressed parameters in the case of ice, which is considered as transversely isotropic in nature (Hearmon[9]). The critical velocity of resonance causing fracture has also been evaluated for different prestressed parameters. In the absence of initial stresses the results have been shown to coincide with the result obtained by Mukherjee[4]. Further, by making the medium to be isotropic, the surface to be smooth and removing the initial stresses, the results have been shown to tally with those of Cole and Huth[1].

The static results derived here for the isotropic elastic half-space without initial stresses also agree with the result given by Timoshenko[10].

FORMULATION

Consider a transversely isotropic elastic half-space $z \geqslant 0$ under uniform initial compressive stresses S_{11} and S_{33} acting along the x- and z-directions, respectively. A normal load F moves with a constant speed v over the surface along the direction of x.

The dynamic equations of motion in an initially stressed medium are given by Biot[11]:

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{31}}{\partial z} + P\frac{\partial \omega_y}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2}, \qquad (1)$$
$$\frac{\partial s_{31}}{\partial x} + \frac{\partial s_{33}}{\partial z} + P\frac{\partial \omega_y}{\partial x} = \rho\frac{\partial^2 w}{\partial t^2},$$

where $P = S_{33} - S_{11}$, and the incremental stress components s_{ij} in terms of displacement components u and w may be written as

$$s_{11} = (c_{11} + P)\frac{\partial u}{\partial x} + (c_{13} + P)\frac{\partial w}{\partial z},$$
$$s_{33} = c_{33}\frac{\partial w}{\partial z} + c_{13}\frac{\partial u}{\partial x}, \qquad (2)$$
$$s_{13} = s_{31} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

c_{ij} being the elastic constants of the medium considered.

The stress equations of motion (1) with the help of (2) become

$$(c_{11} + P)\frac{\partial^2 u}{\partial x^2} + \left(c_{44} + \frac{P}{2}\right)\frac{\partial^2 u}{\partial z^2} + \left(c_{13} + c_{44} + \frac{P}{2}\right)\frac{\partial^2 w}{\partial z\,\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \qquad (3)$$
$$\left(c_{44} - \frac{P}{2}\right)\frac{\partial^2 w}{\partial x^2} + c_{33}\frac{\partial^2 w}{\partial z^2} + \left(c_{13} + c_{44} + \frac{P}{2}\right)\frac{\partial^2 u}{\partial x\,\partial z} = \rho\frac{\partial^2 w}{\partial t^2}.$$

The boundary conditions may be written as

$$\Delta f_z = -F\delta(x - vt) = -\frac{F}{\pi}\int_0^\infty \cos k(x - vt)\,\mathrm{d}k, \qquad (4)$$
$$\Delta f_x = -FR\delta(x - vt) = -\frac{FR}{\pi}\int_0^\infty \cos k(x - vt)\,\mathrm{d}k,$$

where Δf_z, Δf_x are the incremental normal and shear boundary forces, respectively, per unit initial area and are given by[11]

$$\Delta f_z = c_{33}\frac{\partial w}{\partial z} + (c_{13} + S_{33})\frac{\partial u}{\partial x}, \qquad (5)$$
$$\Delta f_x = \left(c_{44} + \frac{P}{2}\right)\frac{\partial u}{\partial z} + \left(c_{44} + \frac{P}{2} - S_{33}\right)\frac{\partial w}{\partial x}.$$

Further, $\delta(x)$ is the Dirac delta function of argument x, and R is the frictional coefficient.

SOLUTION

The steady-state solutions of equations of motion (3) may be assumed to be

$$u = \int_0^\infty A\,e^{-kqz}\cos k(x - vt)\,dk + \int_0^\infty B\,e^{-kqz}\sin k(x - vt)\,dk,$$
$$w = \int_0^\infty C\,e^{-kqz}\cos k(x - vt)\,dk + \int_0^\infty D\,e^{-kqz}\sin k(x - vt)\,dk. \tag{6}$$

Inserting (6) into (3) the following equations for the constants A, B, C, D may be obtained:

$$[\rho v^2 - (c_{11} + P) + (c_{44} + P/2)\,q^2]A - q[c_{13} + c_{44} + P/2]D = 0,$$
$$q[c_{13} + c_{44} + P/2]A + [\rho v^2 - (c_{44} - P/2) + c_{33}q^2]D = 0, \tag{7}$$
$$[\rho v^2 - (c_{11} + P) + (c_{44} + P/2)q^2]B + q[c_{13} + c_{44} + P/2]C = 0,$$
$$q[c_{13} + c_{44} + P/2]B - [\rho v^2 - (c_{44} - P/2) + c_{33}q^2]C = 0.$$

Equations (7) are consistent if

$$q^4 + \left[\frac{\rho v^2 - (c_{44} - P/2)}{c_{33}} + \frac{\rho v^2 - (c_{11} + P)}{c_{44} + P/2} + \frac{(c_{13} + c_{44} + P/2)^2}{c_{33}(c_{44} + P/2)}\right] q^2 + \frac{[\rho v^2 - (c_{11} + P)][\rho v^2 - (c_{44} - P/2)]}{c_{33}(c_{44} + P/2)} = 0. \tag{8}$$

Let q_1^2 and q_2^2 be the two roots of eqn (8). Both q_1 and q_2 will be real if q_1^2, q_2^2 are positive, i.e. if $[\rho v^2 - (c_{11} + P)]\,[\rho v^2 - (c_{44} - P/2)]$ is positive. Hence, v must be either greater than or less than both $[(c_{11} + P)/\rho]^{1/2}$ and $[(c_{44} + P/2)/\rho]^{1/2}$. But if v is greater than both $[(c_{11} + P)/\rho]^{1/2}$ and $[(c_{44} - P/2)/\rho]^{1/2}$, the coefficient of q^2 and the constant term in eqn (8) become positive; therefore both q_1^2 and q_2^2 become negative. Therefore, q_1 and q_2 will be both positive when and only when v is less than both $[(c_{11} + P)/\rho]^{1/2}$ and $[(c_{44} - P/2)/\rho]^{1/2}$.

Then the solution (6) may be written in explicit form as

$$u = \int_0^\infty A_1 e^{-kq_1z}\cos k(x - vt)\,dk + \int_0^\infty A_2 e^{-kq_2z}\cos k(x - vt)\,dk$$
$$+ \int_0^\infty B_1 e^{-kq_1z}\sin k(x - vt)\,dk + \int_0^\infty B_2 e^{-kq_2z}\sin k(x - vt)\,dk,$$
$$w = \int_0^\infty m_1B_1 e^{-kq_1z}\cos k(x - vt)\,dk + \int_0^\infty m_2B_2 e^{-kq_2z}\cos k(x - vt)\,dk \tag{9}$$
$$- \int_0^\infty m_1A_1 e^{-kq_1z}\sin k(x - vt)\,dk - \int_0^\infty m_2A_2 e^{-kq_2z}\sin k(x - vt)\,dk,$$

where

$$m_{1,2} = \frac{(c_{11} + P) - \rho v^2 - (c_{44} + P/2)\,q_{1,2}^2}{(c_{13} + c_{44} + P/2)\,q_{1,2}}.$$

The boundary conditions (4) with (5) and (9) yield

$$A_1[(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1] + A_2[(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2] - \frac{2FR}{\pi k} = 0,$$

$$A_1[m_1q_1c_{33} - (c_{13} + S_{33})] + A_2[m_2q_2c_{33} - (c_{13} + S_{33})] = 0. \quad (10)$$

$$B_1[(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1] + B_2[(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2] = 0,$$

$$B_1[(c_{13} + S_{33}) - m_1q_1c_{33}] + B_2[(c_{13} + S_{33}) - m_2q_2c_{33}] + \frac{F}{\pi k} = 0,$$

from which the constants are obtained as

$$A_1 = \frac{2FR}{\pi k}\left[\frac{m_2q_2c_{33} - (c_{13} + S_{33})}{\Delta^*}\right],$$

$$A_2 = -\frac{2FR}{\pi k}\left[\frac{m_1q_1c_{33} - (c_{13} + S_{33})}{\Delta^*}\right],$$

$$B_1 = -\frac{F}{\pi k}\left[\frac{(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2}{\Delta^*}\right],$$

$$B_2 = \frac{F}{\pi k}\left[\frac{(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1}{\Delta^*}\right],$$

where

$$\Delta^* = [(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1]\,[m_2q_2c_{33} - (c_{13} + S_{33})] - [(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2]\,[m_1q_1c_{33} - (c_{13} + S_{33})].$$

Thus, the incremental stresses in the medium, due to the moving load over the surface of an initially stressed elastic half-space, are

$$s_{13} = \frac{Fc_{44}}{\pi\Delta^*}\left[\int_0^\infty [(m_1 + q_1)\{(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2\}e^{-kq_1z} - (m_2 + q_2)\{(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1\}e^{-kq_2z}]\sin k(x - vt)\,dk + 2R\int_0^\infty [(m_2 + q_2)\{m_1q_1c_{33} - (c_{13} + S_{33})\}e^{-kq_2z} - (m_1 + q_1)\{m_2q_2c_{33} - (c_{13} + S_{33})\}e^{-kq_1z}]\cos k(x - vt)\,dk\right]$$

and

$$s_{33} = \frac{F}{\pi\Delta^*}\left[\int_0^\infty [(c_{13} - m_2q_2c_{33})\{(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1\}e^{-kq_2z} - (c_{13} - m_1q_1c_{33})\{(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2\}e^{-kq_1z}]\cos k(x - vt)\,dk + \int_0^\infty 2R[(m_1q_1c_{33} - c_{13})\{m_2q_2c_{33} - (c_{13} + S_{33})\}e^{-kq_1z} - (m_2q_2c_{33} - c_{13})\{m_1q_1c_{33} - (c_{13} + S_{33})\}e^{-kq_2z}]\sin k(x - vt)\,dk\right].$$

It may be noted that the integrals in the expressions for s_{13} and s_{33} converge for positive values of q_1, q_2, i.e. for values of v less than both $[(c_{11} + P)/\rho]^{1/2}$ and $[(c_{44} - P/2)/\rho]^{1/2}$.

Evaluating the integrals, the expressions for the stresses may be written as

$$s_{13} = \frac{Fc_{44}}{\Delta^*}\left[\left\{\frac{(m_1 + q_1)\,[(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2]}{(x - vt)^2 + q_1^2 z^2} - \frac{(m_2 + q_2)\,[(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1]}{(x - vt)^2 + q_2^2 z^2}\right\}(x - vt) + 2R\left\{\frac{(m_2 + q_2)\,[m_1 q_1 c_{33} - (c_{13} + S_{33})]q_2 z}{(x - vt)^2 + q_2^2 z^2} - \frac{(m_1 + q_1)\,[m_2 q_2 c_{33} - (c_{13} + S_{33})]q_1 z}{(x - vt)^2 + q_1^2 z^2}\right\}\right] \quad (11)$$

and

$$s_{33} = \frac{F}{\pi\Delta^*}\left[\frac{[(2c_{44} - S_{33} - S_{11})m_1 + (2c_{44} + P)q_1]\,(c_{13} - m_2 q_2 c_{33})q_2 z}{(x - vt)^2 + q_2^2 z^2} - \frac{[(2c_{44} - S_{33} - S_{11})m_2 + (2c_{44} + P)q_2]\,(c_{13} - m_1 q_1 c_{33})q_1 z}{(x - vt)^2 + q_1^2 z^2} + 2R\left\{\frac{(m_1 q_1 c_{33} - c_{13})\,[m_2 q_2 c_{33} - (c_{13} + S_{33})]}{(x - vt)^2 + q_1^2 z^2} - \frac{(m_2 q_2 c_{33} - c_{13})\,[m_1 q_1 c_{33} - (c_{13} + S_{33})]}{(x - vt)^2 + q_2^2 z^2}\right\}(x - vt)\right]. \quad (12)$$

Thus, it is seen from expressions (11) and (12) that the stresses generated due to the moving load are functions of the inherent initial stresses (or prestresses) existing in the elastic solid, irrespective of the anisotropic nature of the solid.

In the absence of initial stresses results (11) and (12) coincide with the result obtained by Mukherjee[4].

Further, if we reduce the half-space from an anisotropic to an isotropic medium, i.e. $c_{44} \to \mu$, $c_{13} \to \lambda$, $c_{33} \to (\lambda + 2\mu)$, the developed incremental stresses may be written in nondimensionalized form as

$$s_{13} = \frac{F}{\pi\Delta^*}\left[\left\{\frac{(m_1 + q_1)\,[(1 - \zeta - \zeta')m_2 + (1 + \zeta - \zeta')q_2]}{(x - vt)^2 + q_1^2 z^2} - \frac{(m_2 + q_2)\,[(1 - \zeta - \zeta')m_1 + (1 + \zeta - \zeta')q_1]}{(x - vt)^2 + q_2^2 z^2}\right\}(x - vt) + R\left\{\frac{(m_2 + q_2)\,[(m_1 q_1 - 1)(\alpha^2/\beta^2) + 2(1 - \zeta)]q_2 z}{(x - vt)^2 + q_2^2 z^2} - \frac{(m_1 + q_1)\,[(m_2 q_2 - 1)(\alpha^2/\beta^2) + 2(1 - \zeta)]q_1 z}{(x - vt)^2 + q_1^2 z^2}\right\}\right], \quad (13)$$

$$s_{33} = \frac{F}{\pi\Delta^*}\left[\frac{\{(1-\zeta-\zeta')m_1 + (1+\zeta-\zeta')q_1\}\{(1-m_2q_2)(\alpha^2/\beta^2)-2\}q_2z}{(x-vt)^2+q_2^2z^2}\right.$$

$$-\frac{\{(1-\zeta-\zeta')m_2 + (1+\zeta-\zeta')q_2\}\{(1-m_1q_1)(\alpha^2/\beta^2)-2\}q_1z}{(x-vt)^2+q_1^2z^2}$$

$$+R\left\{\frac{[(m_1q_1-1)(\alpha^2/\beta^2)+2][(m_2q_2-1)(\alpha^2/\beta^2)+2(1-\zeta)]}{(x-vt)^2+q_1^2z^2}\right.$$

$$\left.\left.-\frac{[(m_2q_2-1)(\alpha^2/\beta^2)+2][(m_1q_1-1)(\alpha^2/\beta^2)+2(1-\zeta)]}{(x-vt)^2+q_2^2z^2}\right\}(x-vt)\right], \tag{14}$$

where

$$\Delta^* = [(1-\zeta-\zeta')m_1 + (1+\zeta-\zeta')q_1]\left[(m_2q_2-1)\frac{\alpha^2}{\beta^2}+2(1-\zeta)\right]$$
$$-[(1-\zeta-\zeta')m_2 + (1+\zeta-\zeta')q_2]\left[(m_1q_1-1)\frac{\alpha^2}{\beta^2}+2(1-\zeta)\right],$$

$$\zeta = \frac{S_{33}}{2\mu},\quad \zeta' = \frac{S_{11}}{2\mu},\quad \alpha^2 = \frac{\lambda+2\mu}{\rho},\quad \beta^2 = \frac{\mu}{\rho},$$

$$m_{1,2} = \frac{\alpha^2/\beta^2 - 2(\zeta+\zeta') - v^2/\beta^2 - (1+\zeta-\zeta')q_{1,2}^2}{(\alpha^2/\beta^2+\zeta+\zeta'-1)q_{1,2}}$$

and

$$q_{1,2} = \left[\frac{-b\pm\sqrt{b^2-4c}}{2}\right]^{1/2},$$

in which

$$b = \frac{v^2/\beta^2+\zeta-\zeta'-1}{\alpha^2/\beta^2} + \frac{v^2/\beta^2-\alpha^2/\beta^2-2(\zeta+\zeta')}{1+\zeta-\zeta'} + \frac{(\alpha^2/\beta^2+\zeta-\zeta'-1)^2}{(\alpha^2/\beta^2)(1+\zeta-\zeta')},$$

$$c = \frac{\{v^2/\beta^2-\alpha^2/\beta^2-2(\zeta-\zeta')\}\{v^2/\beta^2+\zeta-\zeta'-1\}}{(\alpha^2/\beta^2)(1+\zeta-\zeta')}.$$

Again, if we remove the initial stresses, i.e. $S_{11} = S_{33} \to 0$, the above results reduce for smooth surface (i.e. $R = 0$) as below:

$$\frac{s_{13}}{F} = -\frac{2(1+q_1^2)q_2}{\Delta^*}\left[\frac{1}{(x-vt)^2+q_2^2z^2} - \frac{1}{(x-vt)^2+q_1^2z^2}\right](x-vt) \tag{15}$$

and

$$\frac{\pi s_{33}}{F} = -\frac{1}{\Delta^*}\left[\frac{(1+q_1^2)^2q_2z}{(x-vt)^2+q_2^2z^2} - \frac{4q_1^2q_2z}{(x-vt)^2+q_1^2z^2}\right], \tag{16}$$

where

$$\Delta^* = (1+q_1^2)^2 - 4q_1q_2,$$
$$q_1^2 = (1-v^2/\beta^2),\qquad q_2^2 = (1-v^2/\alpha^2),$$
$$\alpha^2 = (\lambda+2\mu)/\rho,\qquad \beta^2 = \mu/\rho.$$

Using the notations $\bar{x} = x - vt$, $C_L = \alpha$, $C_T = \beta$, $M_L = v/\alpha$, $M_T = v/\beta$, $\beta_L^2 = (1 - M_L^2)$, $\beta_T^2 = (1 - M_T^2)$, $r_L^2 = (\bar{x}^2 + \beta_L^2 z^2)$, $r_T^2 = (\bar{x}^2 + \beta_T^2 z^2)$, $\theta_L = \tan^{-1}(\beta_L z \sqrt{x})$, $\theta_T = \tan^{-1}(\beta_T z \sqrt{x})$, results (13) and (14) are seen to coincide with the results obtained by Cole and Huth([1], page 434).

From (11) and (12), the stresses at $x = vt$, i.e. at the point directly below the load, may be obtained as

$$s_{13} = \frac{FR}{\pi z \Delta^*}\left[\frac{(m_2 + q_2)\,[m_1 q_1 N - (1 + 2\eta_3)]}{q_2} - \frac{(m_1 + q_1)\,[m_2 q_2 N - (1 + 2\eta_3)]}{q_1}\right], \quad (17)$$

$$s_{33} = \frac{F}{\pi z \Delta^*}\left[\frac{(1 - m_2 q_2 N)\,[(1 - \eta_3' - \eta_1)m_1 + (1 + \eta_3' - \eta_1)q_1]}{q_2} - \frac{(1 - m_1 q_1 N)\,[(1 - \eta_3' - \eta_1)m_2 + (1 + \eta_3' - \eta_1)q_2]}{q_1}\right], \quad (18)$$

where

$$\Delta^* = [m_2 q_2 N - (1 + 2\eta_3)]\,[(1 - \eta_3' - \eta_1)m_1 + (1 + \eta_3' - \eta_1)q_1] - [m_1 q_1 N - (1 + 2\eta_3)]\,[(1 - \eta_3' - \eta_1)m_2 + (1 + \eta_3' - \eta_1)q_2],$$

in which $N = c_{33}/c_{13}$, $\eta_3 = S_{33}/2c_{13}$, $\eta_3' = S_{33}/2c_{44}$ and $\eta_1 = S_{11}/2c_{44}$.

Static case

For the static case we consider $v = 0$. By making the medium isotropic and removing the initial stresses, we get the following static results:

$$s_{13} = -\frac{2F}{\pi z}\frac{x}{(x^2 + z^2)^{1/2}}\frac{z^3}{(x^2 + z^2)^{3/2}}$$

and

$$s_{33} = -\frac{2F}{\pi z}\frac{z^4}{(x^2 + z^2)^2}.$$

The above results tally with the well-known static result obtained by Timoshenko([10], page 84).

NUMERICAL RESULTS

The stresses as given in (17) and (18) have been calculated in case of ice. Following Hearmon[9], the elastic constants for ice at −16°C are

$$\begin{aligned} c_{11} &= 1.36 \times 10^{11}\ \text{dyn/cm}^2, \\ c_{33} &= 1.46 \times 10^{11}\ \text{dyn/cm}^2, \\ c_{44} &= 0.32 \times 10^{11}\ \text{dyn/cm}^2, \\ c_{12} &= 0.67 \times 10^{11}\ \text{dyn/cm}^2, \\ c_{13} &= 0.52 \times 10^{11}\ \text{dyn/cm}^2, \end{aligned} \quad (19)$$

Table 1. $v = 100$ m/sec

$z = 1$ m	η_3	η_3'	η_1	$(s_{33}/F)_{x=vt}$	$(s_{13}/FR)_{x=vt}$
	0.2	0.3	0.5	-5.732×10^{-3}	1.779×10^{-3}
	0.5	0.8	0.2	-1.112×10^{-2}	1.529×10^{-3}
	−0.2	−0.3	−0.5	-7.219×10^{-3}	-0.915×10^{-4}
	0.0	0.0	0.5	-5.133×10^{-3}	1.899×10^{-3}
	0.2	0.3	−0.5	-1.160×10^{-2}	1.967×10^{-3}
	0.5	0.8	0.0	-6.057×10^{-2}	4.437×10^{-3}
	0.0	0.0	0.0	-7.289×10^{-3}	0.132×10^{-4}
$z = 5$ m					
	0.2	0.3	0.5	-1.147×10^{-3}	3.558×10^{-4}
	0.5	0.8	0.2	-2.224×10^{-3}	3.055×10^{-4}
	−0.2	−0.3	−0.5	-1.444×10^{-3}	-0.382×10^{-5}
	0.0	0.0	0.5	-1.027×10^{-3}	3.800×10^{-4}
	0.2	0.3	−0.5	-2.321×10^{-3}	2.762×10^{-3}
	0.5	0.8	0.0	-1.211×10^{-3}	0.887×10^{-3}
	0.0	0.0	0.0	-1.458×10^{-3}	0.255×10^{-5}

Table 2. $v = 200$ m/sec

$z = 1$ m	η_3	η_3'	η_1	$(s_{33}/F)_{x=vt}$	$(s_{13}/FR)_{x=vt}$
	0.2	0.3	0.5	-5.748×10^{-3}	1.865×10^{-3}
	0.5	0.8	0.2	-1.124×10^{-2}	1.544×10^{-3}
	−0.2	−0.3	0.5	1.408×10^{-2}	1.762×10^{-2}
	0.0	0.0	0.5	-5.147×10^{-3}	1.901×10^{-3}
	0.2	0.3	−0.5	-4.657×10^{-3}	1.974×10^{-2}
	0.5	0.8	0.0	-1.262×10^{-2}	1.242×10^{-2}
	0.0	0.0	0.0	-7.306×10^{-3}	0.147×10^{-4}
$z = 5$ m					
	0.2	0.3	0.5	-1.149×10^{-3}	0.373×10^{-3}
	0.5	0.8	0.2	-2.240×10^{-3}	0.248×10^{-3}
	−0.2	−0.3	0.5	2.817×10^{-3}	3.523×10^{-3}
	0.0	0.0	0.5	-1.029×10^{-3}	3.147×10^{-4}
	0.2	0.3	−0.5	-3.948×10^{-3}	-0.893×10^{-3}
	0.5	0.8	0.0	-2.524×10^{-3}	2.484×10^{-3}
	0.0	0.0	0.0	-1.461×10^{-3}	0.293×10^{-5}

and

$$\rho = 0.917 \text{ g-cm}^3.$$

The numerical values of s_{13}/FR and s_{33}/F at points directly below the load are calculated for various prestressed parameters and for different z. The values of v taken are 100 m/sec and 200 m/sec. The numerical results are in Tables 1 and 2.

DISCUSSION

It is observed that Δ^* occurs in the denominator of the expressions for the stress components. Hence, in case $\Delta^* = 0$ there will be a high stress concentration which will lead the material towards fracture. Thus $\Delta^* = 0$ will determine the velocity v of the moving load F at which the fracture will take place. The presence of η_1, η_3 and η_3' in the equation $\Delta^* = 0$ shows the influence of prestresses in the medium on the formation of fracture.

From $\Delta^* = 0$ the values of $v/\sqrt{c_{44}/\rho}$ have been calculated for ice using the same elastic constants as given in (19) for different sets of prestress parameters η_1, η_3 and η_3', and the following results have been obtained.

η_3	η_3'	η_1	$v^*/\sqrt{c_{44}/\rho}$
0	0	0	0.9584
0	0	0.5	1.2125
−0.2	−0.3	0	0.9114
−0.2	−0.3	−0.5	0.6365
0.2	0.3	−0.5	0.3816

The first result is for the material free from initial stresses, and in this case the velocity at which the fracture will take place is very close to the velocity of Rayleigh wave propagation in the transversely isotropic half-space. This was expected too. The result shows that if the tensile prestress in the x-direction is increased, the fracture occurs at higher velocity of the moving load, whereas if the prestress in the x-direction is compressible, then lower velocity can create fracture. Further, it has been observed during calculation that if the tensile initial stress in the z-direction is higher, then there is less possibility of fracture.

From the numerical results, specifically, it may be said that the development of the incremental normal and the shearing stresses induced by the initial tensile stresses along the z-direction increases. However, the normalized initial tensile stress along the x-direction increases the development of shearing stress but decreases the normal stress.

The presence of either the tensile initial stress or compressive initial stress in both directions increases the shearing stress but decreases the normal stress. Also, it may be noted that the development of shearing stress in the case of initial compressive stress is comparatively less than the tensile case, but for normal stress the behaviour is just the opposite.

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